Oxford Cambridge and RSA

Wednesday 22 June 2022 – Afternoon

AS Level Further Mathematics B (MEI)

Y415/01 Mechanics b

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

DC (ST/CGW) 301221/5

Answer **all** the questions.

1 A small smooth ring of mass 0.5 kg is travelling round a smooth circular wire, with centre O and radius 0.8 m. The circle of wire is in a horizontal plane.

The speed of the ring, $v\,\mathrm{m\,s^{-1}}$, at time t s after passing through a point A on the wire is given by $v = 0.2t^2 + 0.4t + 0.1$.

(a) Find the angular speed of the ring 5 seconds after it passes through A. **[2]**

(b) Find the distance the ring travels along the wire in the first second after passing through A. **[2]**

At time T s after the ring passes through A the magnitude of the force exerted on the ring by the wire is 6.4 N. You may assume that any forces acting on the ring other than the force exerted on the ring by the wire and gravity can be ignored.

(c) **(i)** Determine the value of T. **[3]**

(ii) Hence find the tangential acceleration of the ring at this time. **[2]**

2 A light elastic string has natural length a and modulus of elasticity kmg, where $k > 2$. One end of the string is attached to a fixed point O. A particle P of mass m is attached to the other end of the string. P is held at rest a distance $\frac{3}{2}a$ vertically below O.

At time t after P is released, its vertical distance below O is y.

(a) Show that, while the string is in tension, the equation of motion of P is given by the differential equation $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = (k+1)g - \frac{kg}{a}y$. **[3]**

A student transforms the differential equation in part **(a)** into the standard SHM equation $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x$.

(b)
- Find an expression for x in terms of y, k and a.
- Find an expression for ω in terms of k, a and g. **[3]**

3 **Fig. 3.1** shows the curve with equation $y = x^2 + 3$. The region R, shown shaded, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$. A uniform solid of revolution S is formed by rotating the region R through 2π about the x-axis.

The volume of S is $\frac{202}{5}\pi$.

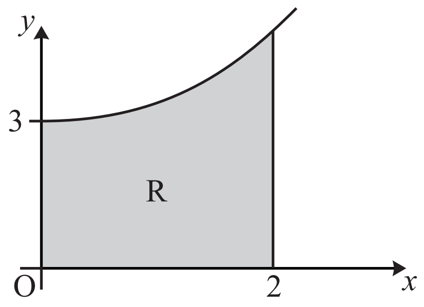


Fig. 3.1

(a) **In this question you must show detailed reasoning.**

Show that the x-coordinate of the centre of mass of S is $\frac{395}{303}$. **[4]**

S is fixed to a cylinder of base radius 3 units and height 2 units to form the uniform solid D. The smaller circular face of S is joined to the top circular face of the cylinder, as shown in **Fig. 3.2**.

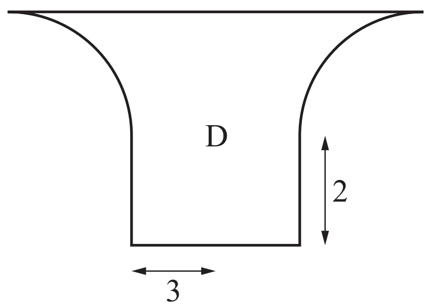


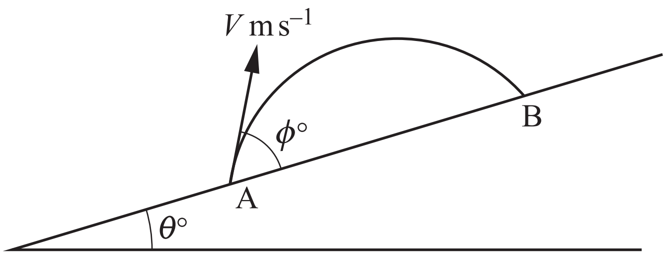
Fig. 3.2

(b) Find the distance of the centre of mass of D from its smaller circular face. **[4]**

D is placed with its smaller circular face in contact with a rough plane which is inclined at an angle of 30° to the horizontal. It is given that D does not slip.

(c) Determine whether D topples. **[2]**

4 A plane is inclined at an angle $\theta°$ to the horizontal. A particle is projected from a point A on the plane with speed $V\,\mathrm{m\,s^{-1}}$ in a direction making an angle of $\phi°$ with a line of greatest slope of the plane. The particle lands at a point B on the plane, as shown in the diagram, and the time of flight is T seconds.



(a) By considering the motion of the particle perpendicular to the plane, show that $T = \dfrac{2V\sin\phi}{g\cos\theta}$. **[4]**

Consider the case when $\theta = 30$, $\phi = 25$ and $V = 20$.

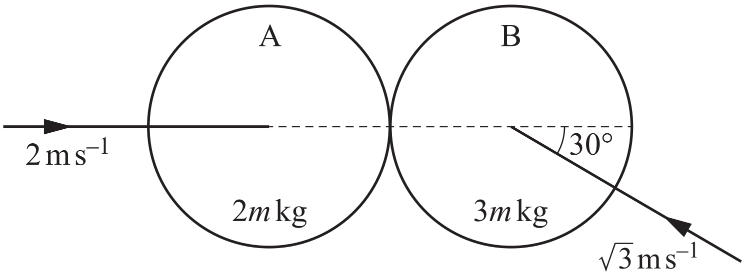
(b) (i) Calculate the distance AB. **[3]**

(ii) State, with reasons but without any detailed calculations, what effect each of the following actions would have on the distance AB.

- Increasing V while leaving θ and ϕ unchanged.
- Increasing ϕ while leaving θ and V unchanged. **[3]**

5 Two small uniform discs, A of mass $2m$ kg and B of mass $3m$ kg, slide on a smooth horizontal surface and collide obliquely with smooth contact.

Immediately before the collision, A is moving towards B along the line of centres with speed $2\,\text{m}\,\text{s}^{-1}$ and B is moving towards A with speed $\sqrt{3}\,\text{m}\,\text{s}^{-1}$ in a direction making an angle of 30° with the line of centres, as shown in the diagram.



(a) Explain how you know that the motion of A will be along the line of centres after the collision. **[1]**

(b) • Determine the maximum possible speed of A after the collision.

• Find the value of the coefficient of restitution in this case. **[7]**

(c) • Determine the minimum possible speed of B after the collision.

• Find the value of the coefficient of restitution in this case. **[3]**

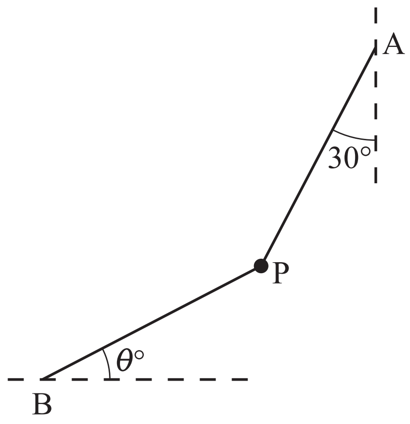
When the speed of B after the collision is a minimum, the loss of kinetic energy in the collision is 1.4625 J.

(d) Determine the value of m. **[4]**

6 Two identical light elastic strings, each of length l and modulus of elasticity λmg are attached to a particle P of mass m.

The other end of the first string is attached to a fixed point A, and the other end of the second string is attached to a fixed point B. The points A and B are such that A is above and to the right of B and both strings are taut.

The string attached to A makes an angle of 30° with the vertical, and the string attached to B makes an angle of $\theta°$ with the horizontal, as shown in the diagram.



The system is in equilibrium in a vertical plane. The extension of the string attached to A is $0.9l$ and the extension of the string attached to B is $0.5l$.

(a) Explain how you know that APB is **not** a straight line. **[1]**

(b) Show that the elastic potential energy stored in string AP is $kmgl$, where the value of k is to be determined correct to **3** significant figures. **[9]**

END OF QUESTION PAPER

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